

## Modelling the optimal phosphate fertiliser and soil management strategy for crops

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# Use of a coupled soil-root-leaf model to optimise phosphate fertiliser use efficiency in barley

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## Abstract: (200192/200)

*Aims* Phosphorus (P) is an essential nutrient necessary for maintaining crop growth, however, it's often used inefficiently within agroecosystems, driving industry to find new ways to deliver P to crops sustainably. We ~~consider a precision agriculture approach which~~ aim to combines traditional soil and crop measurements with climate-driven mathematical models, ~~that can~~ to optimise the timing and placement of fertiliser applications.

*Methods* The whole plant crop model combines an above-ground leaf model with an existing spatially explicit below-ground root-soil model to estimate plant P uptake and leaf mass. We let P-dependent photosynthesis estimate carbon (C) mass, which in conjunction with temperature sets the root-growth-rate.

*Results* The addition of the leaf model achieved a better estimate of two sets of barley field trial data for ~~leaf mass and~~ plant P uptake, compared with just the root-soil model alone. Furthermore,

23 discrete fertiliser placement increases plant P uptake by up to 10% in comparison to incorporating  
24 fertiliser.

25 *Conclusions* By capturing essential plant processes we are able to accurately simulate P and C use  
26 and water and P movement during a cropping season. The powerful combination of mechanistic  
27 modelling and experimental data allows physiological processes to be quantified accurately and  
28 useful agricultural predictions for site specific locations.

29 **Keywords** Mathematical modelling, phosphate, phosphorus, fertiliser strategy, barley field study,  
30 above and below ground

## 31 **Introduction**

32 The world-wide production of food has increased due to the demands of an ever expanding global  
33 human population (Brown, 2012). Due to the lack of land available for agricultural expansion, there  
34 is a need to increase yields sustainably by manipulating the existing environment in which crops are  
35 grown, and breeding more resource efficient crops. Resource management for arable farming  
36 systems is critical to the survival of the human population and large amounts of money and time are  
37 needed to elicit the appropriate improvements (Conway and Barbier, 1990).

38 Phosphorus (P) is one of the essential nutrients required for plant growth and plays an important  
39 role in photosynthesis, respiration, and seed and fruit production.

40 We are interested in how crops grow and survive in low P environments and how fertiliser and soil  
41 cultivation methods are influencing crop performance. A number of studies have considered the  
42 response of adding different amounts and rates of fertiliser P; in some soils large effects are seen  
43 whereas no effect is seen in others (Bolland and Baker, 1998; Kuchenbuch and Buczko, 2011;  
44 Valkama et al., 2011). There are many ways one could apply P to soils; for example incorporating  
45 (also known as broadcasting, involves an even spreading of P on top of the soil), placing (also known  
46 as banding, involves injecting P into the soil nearer the rooting zone either in row or between rows)

47 or as a coating on seeds. Studies have shown that injecting fertiliser into the soil nearer to the root  
48 zone (placing) increases plant P uptake compared to incorporated P (Randall and Hoeft, 1988; Lohry,  
49 1998; Owusu-Gyimah *et al.*, 2013). In addition, studies have been conducted to estimate the  
50 differences in soil cultivation methods on plant P uptake; for example, conventional plough *versus*  
51 minimum tillage (also considering gene variation, George *et al.*, 2011). The idea behind ploughing is  
52 to turn over or mix the top 25 cm of soil to loosen the soil for seeding, bury any existing crop  
53 residues or weeds, and to provide a good distribution of nutrients for the coming crop. This is in  
54 contrast to minimum tillage which enhances topsoil stability against erosion, retains moisture and  
55 reduces crop establishment costs, but segregates P content with depth and can leave 30% of crop  
56 residue on the soil surface.

57 Due to the rising cost of fertilisers and agricultural machinery, crop production has become a multi-  
58 objective optimisation problem to minimise multiple costs while trying to maximise the crop yield  
59 and environmental impact of fertilisers. This is a complex problem due to varying climatic conditions,  
60 an abundance of technological machines, and availability of more data concerning the states of  
61 fields than ever before. Precision agriculture is an emerging field involved with combining the  
62 newest technologies to the farming industry, ranging from unmanned drone maps of fields to  
63 computer-assisted tractors (Blackmore, 2014). This new technology is enabling automated real time  
64 decision making, applying the most effective treatment to crops at the best time for the best price.  
65 Mathematical models, supported by experimental data, are needed to help predict best decisions in  
66 the short term, and also strategically, to optimise between possible future options. Whilst such  
67 models are ~~seldom-not always commercially usedemployed-at-present~~, their potential capabilities  
68 are attractive, given that field-scale experiments are both costly and time-consuming, and  
69 integration and dissemination of their empirical results is challenging (Selmants and Hart, 2010;  
70 Jeuffroy *et al.*, 2012; Sylvester-Bradley, 1991).

71 A plethora of models exist that describe the processes involved in plant growth and the behaviour of  
72 nutrients and water in the soil. Each model has its own unique assumptions and is generally targeted  
73 at specific scientific problems within the area of agriculture. For example, Greenwood *et al.* (2001)  
74 developed a dynamic model (PHOSMOD) for the effects of soil P and fertiliser P on crop growth, P  
75 uptake and soil P in arable cropping; Jones *et al.* (2003) describe a decision support system for  
76 agrotechnology transfer (DSSAT) which focuses on average plant-environment interactions; and  
77 Keating *et al.* (2003) review an agricultural production systems simulation (APSIM) developed in  
78 CISRO, Australia which deals with water, N, P, pH, erosion and management issues. At the beginning  
79 of the 21<sup>st</sup> century, modelling 3D architectures of plant roots (RootBox, ROOTMAP, SimRoot,  
80 RootTyp, SPACSYS, R-SWMS) has become popular (Dunbabin *et al.*, 2013). In addition, two research  
81 groups that model above ground 3D plant structures, Prunskiewicz Algorithmic Botany group at  
82 the University of Calgary and the Andrieu group (ADEL-wheat model), both use L systems to simulate  
83 the above ground structure of wheat plants. L systems, introduced by Lindenmayer in 1968,  
84 represent a string of production rules that are used to create geometric structures, ideal for plant  
85 development. However all these models do not describe the root-soil interaction explicitly and do  
86 not fully integrate functions that occur above ground with ones that occur below ground. Therefore  
87 plants of the same genotype are represented alike and phenotypic differences cannot be observed.  
88 We hope to address some of these problems by creating a model that links the above and below  
89 ground processes in such a way that they rely on one another. Our whole crop model is based on a  
90 below ground plant-soil interaction model (Roose and Fowler, 2004b; Heppell *et al.*, 2015) coupled  
91 with an above ground leaf growth model based on the seminal work of Thornley (1995).

92 Here we describe a whole crop model that includes a below-ground root model and an above-  
93 ground leaf model and which is validated against experimental data on barley with a varying P  
94 fertiliser scenario analysis. The development of the model is seen as a step-change in our  
95 computational capability to help predict soil P supply, crop P uptake patterns and fertilizer  
96 requirements.

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97 **Materials and Methods**

98 **Experimental data**

99 Two barley field trial data sets are used, consisting of leaf mass and plant P uptake values at  
100 different growth stages (GS31, GS45 and GS91 for spring barley; GS39 and GS92 for winter barley).

101 The experimental data includes different rates of P application (0, 5, 10, 20, 30, 60, 90 kg P ha<sup>-1</sup> for  
102 spring barley; 0 15, 30, 60, 90, 120 kg P ha<sup>-1</sup> for winter barley) and both sites were classified with an

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103 Olsen P index 1 soil. -The protocol for this is described in Heppell *et al.* (2015). In addition, we use  
104 the climate data, from the UK Met office Integrated Data Archive System (MIDAS), to accompany the  
105 spring barley (Inverurie, Scotland) and winter barley (Cambridge, England) data sets for the specific  
106 fields in the trial. The climate data consists of daily values for mean temperature (°C), rainfall (mm),  
107 wind speed (m s<sup>-1</sup>) and humidity (%).

108 **Modelling the whole crop**

109 In this paper we extend a root-soil model (Roose and Fowler, 2004b; Heppell *et al.*, 2015) which  
110 estimates plant P uptake, with an above ground model which estimates leaf mass (based on  
111 Thornley, 1995), to produce a whole crop model. We first describe the root-soil model (hereafter  
112 called the root model), followed by the leaf model and then our coupling process to create a whole  
113 crop model.

114 **Root and soil model**

115 To model the root system we follow the same approach as described in Roose *et al.* (2004b) and  
116 Heppell *et al.* (2015) by modelling two orders of root branches only (main and first order branches).  
117 First order roots branch off the main order roots at a given density ( $\psi_1$ ), branching angle ( $\theta$ ), and  
118 each order of roots has a given maximum length and radius ( $L_0$ ,  $L_1$  and  $a$ ,  $a_1$  for main and first order  
119 roots, respectively). As in Roose *et al.* (2004b) and Heppell *et al.* (2015) we let the root growth slow

down as the root becomes longer. Following Heppell *et al.* (2015) we also let the root growth rate ( $r$ ) be dependent upon temperature  $T$ ,

Eqn. 1 
$$\frac{\partial l_i}{\partial t} = r(T(t)) \left(1 - \frac{l_i}{L_i}\right),$$

where  $l_i$  is the current length of an order  $i$  root and  $L_i$  is the maximum length of an order  $i$  root.

The root-soil model is described by the following two equations for water saturation (Eqn. 2) and P (Eqn. 3) concentration respectively,

Eqn. 2 
$$\phi \frac{\partial S}{\partial t} = \nabla \cdot [D_0 D(S) \nabla S - K_S k(S) \hat{\mathbf{k}}] - F_w(S, z, t),$$

Eqn. 3 
$$\frac{\partial}{\partial t} [(b + \phi S)c] + \nabla \cdot [c\mathbf{u}] = \nabla \cdot [D_f \phi^d S^d \nabla c] - F(c, z, t),$$

where the water flux in the soil,  $\mathbf{u}$ , is given by Darcy's law,

Eqn. 4 
$$\mathbf{u} = -D_0 D(S) \nabla S + K_S k(S) \hat{\mathbf{k}}.$$

In the above equations  $S$  is the relative water saturation given by  $S = \phi_1 / \phi$ ,  $\phi_1$  is the volumetric water content, and  $\phi$  is the porosity of the soil.  $D_0$  ( $\text{cm}^2 \text{ day}^{-1}$ ) and  $K_S$  ( $\text{cm day}^{-1}$ ) are the parameters for water 'diffusivity' and hydraulic conductivity, respectively (Van Genuchten, 1980).  $D(S)$  and  $K(S)$  characterize reduction in water 'diffusivity' and hydraulic conductivity in response to the relative water saturation decrease, where the functional forms for partially saturated soil are given by Van Genuchten (1980).  $\hat{\mathbf{k}}$  is the vector pointing vertically downwards from the soil surface and  $F_w$  is the water uptake by the plant root system per unit volume of soil as given by Roose and Fowler (2004a).

For the total P conservation (Eqn. 3),  $c$  is the P concentration in soil pore water,  $b$  is the soil buffer power characterising the amount of P bound to the soil particle surfaces,  $D_f$  is the P diffusivity in free water and  $d$  is an impedance factor;  $1 \leq d \leq 3$  (Barber, 1984; Nye and Tinker, 1977).  $F(c, S, t)$  describes the rate of plant P uptake by a root branching structure (Roose *et al.*, 2001). Both  $F_w$  and  $F$

are affected by the spatially and temporally evolving root structure. Water is only taken up by the main order roots while  $P$  is taken up by all roots; see Roose and Fowler (2004b) for details of the derivation. The equation for  $F_w$  is given by,

Eqn. 5 
$$F_w = \frac{2\pi a_1 k_r + (2\pi a_1 k_r k_z)^{\frac{1}{2}} \psi_1(z)}{\pi(a + L_1 \cos \theta)^2} [-p_c f(S) - p_r],$$

where  $\psi_1$  is the density of first order roots on the main order roots,  $a_1$  is the first order root radius,  $a$  is the main order root radius,  $L_1$  is the maximum length of the first order branches,  $\theta$  is the angle between the main root and the first order branches,  $k_r$  is the root radial water conductivity parameter ( $\text{m s}^{-1} \text{Pa}^{-1}$ ),  $k_z$  is the root axial hydraulic conductivity calculated using Poiseuille law ( $\text{m}^4 \text{Pa}^{-1} \text{s}^{-1}$ ),  $p_c$  (Pa) is a characteristic suction pressure determined from experimental data for different types of soil,  $f(S) = (s^{-1/m} - 1)^{1-m}$ , where  $m$  is the Van Genuchten soil suction parameter (where  $0 < m < 1$ ), and  $p_r$  is the root internal xylem pressure (Pa).

Root internal xylem pressure ( $p_r$ ) is calculated by balancing radial and axial fluid fluxes inside the root, i.e. after Roose and Fowler (2004a) we have,

Eqn. 6 
$$2\pi a k_r (-p_c f(S) - p_r) = -k_z \frac{\partial^2 p_r}{\partial x^2},$$

with two boundary conditions; an impermeable root tip (Eqn. 7) and a root internal pressure ( $P$ ) at the base of the zero order root (Eqn. 8),

Eqn. 7 
$$\frac{\partial p_r}{\partial x} = 0 \text{ at } x = L,$$

Eqn. 8 
$$p_r = P \text{ at } x = 0,$$

where  $P$  is a function of temperature ( $T$ ), humidity ( $H$ ) and a base line pressure ( $p_r^0$ ) for fitting parameters  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$  (see Heppell *et al.*, 2014 for the procedure to estimate them), i.e.

Eqn. 9 
$$P = (p_r^0 + \lambda_3) + \lambda_1 T + \lambda_2 H.$$



163 The rate of plant P uptake is given by,

164 Eqn. 10 
$$F(c, z, t) = \frac{F_0 + F_1}{\pi(a + L_1 \cos \theta)^2},$$

165 where  $F_0$  and  $F_1$  are the uptake rates for zero and first order roots derived in Roose *et al.* (2004b).

166 The boundary conditions to accompany Equations 1 and 2 include a soil surface boundary condition  
167 for water,

168 Eqn. 11 
$$-D_o D(S) \frac{\partial S}{\partial z} + K_S k(S) = W_{dim} \text{ at } z = 0.$$

169  $W_{dim}$  (the flux of water into the soil) is dependent upon rainfall ( $R$ ), humidity ( $H$ ), temperature ( $T$ ),  
170 wind speed ( $WS$ ) and a constant ( $E$ ) which sets a base line flux i.e.

171 Eqn. 12 
$$W_{dim} = \delta R + \alpha H + \beta T + \gamma WS + E,$$

172 for fitting parameters  $\delta$ ,  $\alpha$ ,  $\beta$  and  $\gamma$  (see Heppell *et al.*, 2014 for how these values were estimated).

173 In addition, we have a boundary condition for the concentration of P ( $c$ ) at the soil surface,

174 Eqn. 13 
$$-D_f \phi^d S^d \frac{\partial c}{\partial z} + W_{dim} c = 0 \text{ at } z = 0, \text{ for } t > 0.$$

175 We set a zero flux at the bottom of the soil ( $l_w$ ) for both P and water,

176 Eqn. 14 
$$-D_o D(S) \frac{\partial S}{\partial z} + K_S k(S) = 0 \text{ at } z = l_w,$$

177 Eqn. 15 
$$-D_f \phi^d S^d \frac{\partial c}{\partial z} = 0 \text{ at } z = l_w.$$

178 The initial state of P concentration and water saturation in the soil is given where possible by the  
179 initial soil data for the spring and winter barley experimental sites. A uniform water saturation  
180 profile is initially set at  $S = 0.3$  for the two experimental sites; however for the initial P  
181 concentration ( $c_0(z)$ ) we consider two different cases; (1) a uniform concentration and (2) an  
182 exponentially decaying concentration:

183 Eqn. 16

$$\begin{aligned} (1) \quad c_0(z) &= c_A & \text{at } t = 0, \forall z \\ (2) \quad c_0(z) &= A_1 e^{-B_1 z} & \text{at } t = 0, \forall z' \end{aligned}$$

184 where  $c_A$  is set to 16 mg P l<sup>-1</sup>,  $A_1$  is the P concentration at the top of the soil (23 mg P L<sup>-1</sup>) and  $B_1$  is  
 185 the strength of the decay in the concentration of P (0.345). The initial P concentration values ( $C_A$ ,  
 186  $A_1$  and  $B_1$ ) come from a best fit to the data sets in Heppell et al., (2015) and are both classified as an  
 187 Olsen P index 1 soil (Defra, 2010). To reflect the different fertiliser scenarios being used at each field  
 188 site a set amount of P ( $P_1$ ) (0-120 kg P ha<sup>-1</sup>) was either applied at the surface ( $z = 0$ ) (P broadcast) or  
 189 at a set depth below the soil ( $D_1$ ) (P placement).

190 Eqn. 17

$$\begin{aligned} &c = c_0(z) + H(z), \\ (\text{broadcast}) \quad &H(z) = P_1 & \text{at } t = 0, \quad z = 0 \\ (\text{placement}) \quad &H(z) = P_1 & \text{at } t = 0, \quad z = D_1 \\ (\text{else}) \quad &H(z) = 0 & \text{at } t = 0 \quad \forall z \end{aligned}$$

#### 191 Leaf growth model

192 We have altered a compartmental model developed by Thornley (1995) to describe leaf mass  $M_L$   
 193 (kgL), leaf C  $M_C$  (kgC) and leaf P  $M_P$  (kgP) as well as the concentration of free C [ $C$ ] =  $M_C/M_L$  (kgC  
 194 kgL<sup>-1</sup>) and free P [ $P$ ] =  $M_P/M_L$  (kgP kgL<sup>-1</sup>) dynamics within the leaves. The leaf model takes into  
 195 account non-linear dynamics of formation of leaf litter and leaf self-shading. Additionally we have  
 196 made photosynthesis dependent upon P content in the plant (Foyer and Spencer, 1986, Wissuwa *et*  
 197 *al.*, 2005) and we have altered the leaf growth term,  $G_{sh}$ , which was dependent on [C] and [P], to  
 198 also depend upon ~~the~~ air temperature ( $A_T$ ), ~~for the winter barley but not spring barley, as well as [C]~~  
 199 ~~and [P]. We don't let air temperature affect spring barley as the growing season is much shorter~~  
 200 ~~compared to winter barley and it is not needed for a good fit to the experimental data.~~ The  
 201 governing equations are given below and are represented in a flow diagram on Figure 1, i.e., we  
 202 have

203 Eqn. 18

$$\frac{\partial M_L}{\partial t} = \overbrace{\hat{G}_{sh}}^{\text{Leaf growth rate}} - \overbrace{\frac{K_{litt}}{1 + \frac{K_{mlitt}}{M_L}} M_L}^{\text{Leaf metabolism/litter}},$$

$$\text{Eqn. 19} \quad \frac{\partial M_C}{\partial t} = \frac{\text{Production of C from photosynthesis}}{\varepsilon k_1 [P]} - \frac{\text{Use of C for L growth}}{f_c \widehat{G_{sh}}} - \frac{\text{Output of C to phloem}}{\beta_c [C]},$$

$$\text{Eqn. 20} \quad \frac{\partial M_P}{\partial t} = \frac{\text{Use of P for L growth}}{-f_p \widehat{G_{sh}}} + \frac{\text{Input of P from xylem}}{F(c, z, t)} - \frac{\text{Output of P to phloem}}{\beta_p [P]} - \frac{\text{Use of P to produce C}}{k_p \varepsilon [P] k_1},$$

where,

$$\text{Eqn. 21} \quad G_{sh} = k_G M_L [C] [P] \frac{A_T^{s_1}}{s_2^{s_1} + A_T^{s_1}},$$

$$\text{Eqn. 22} \quad \varepsilon = \frac{k_C M_L}{\left(1 + \frac{M_L}{k_M}\right) \left(1 + \frac{[C]}{J_C}\right)},$$

where  $k_g$  is the leaf growth rate,  $K_{litt}$  is the litter rate,  $K_{mlitt}$  is the litter Michaelis-Menten constant,  $K_C$  is the photosynthesis rate,  $k_M$  is the constant accounting for the leaf self-shading,  $J_C$  is the C product inhibition constant,  $f_c$  is the fraction of total C used for leaf growth,  $f_p$  is the fraction of total P used for leaf growth,  $k_1$  is the amount of P used for photosynthesis,  $k_p k_1$  is the P loss due to photosynthesis,  $\beta_c$  is the rate of C output from the xylem to the phloem,  $\beta_p$  is the rate of P output to the phloem,  $F(c, z, t)$  is the rate of P entry from the xylem (Eqn. 10) and  $s_1$  and  $s_2$  are fitting parameters. Initial values for the leaf ( $M_L$ ), C ( $M_C$ ) and P ( $M_P$ ) mass are  $1 \times 10^{-4}$ , 0 and  $1 \times 10^{-7}$  kg respectively.

### Whole crop model

In order to provide feedback between the root model and leaf model, we allow C mass to affect the root growth rate. Increasing C mass will increase root growth which in turn will increase plant P uptake. Through the process of photosynthesis, increasing plant P uptake will also increase C mass, thus creating a positive feedback loop.

The order  $i$  root growth rate is now dependent on C as well as temperature, therefore we replace

Eqn. 1 with,

224 Eqn. 23 
$$\frac{\partial l_i}{\partial t} = r(T, C) \left(1 - \frac{l_i}{L_i}\right),$$

225 where the rate of growth  $r(T, C)$  is given by a function of temperature multiplied by a function of C  
 226 ( $r(T, C) = f(C)g(T)$ ),

227 Eqn. 24 
$$f(C) = \frac{\alpha_c M_c}{\gamma_c + M_c},$$

228 Eqn. 25 
$$g(T) = \begin{cases} 0 & T \leq 5^\circ C \\ A(T - 5) & T > 5^\circ C \end{cases}$$

229 where  $\gamma_c$  is the mass of C when the root system is at half its maximum size,  $\alpha_c$  is the strength of the  
 230 C effect and  $A$  is a fitting parameter determining the strength of temperature dependence on root  
 231 growth rate. Below critical temperature ( $5^\circ C$ ) there is no root growth and this reflects cold periods  
 232 over the winter (Sylvester-Bradley *et al.*, 2008).

### 233 Fitting process Calibration

234 The parameter list for the models above is given in Table 1. A subset of these parameters are fitted  
 235 to the experimental data and their values can be seen in Table 2. To begin the fitting  
 236 procedure calibration process, the leaf model is first fit against the experimental leaf mass data, by  
 237 6 parameters ( $\beta_c, k_1, f_c, f_p$  for spring barley and in addition  $s_1$  and  $s_2$  for winter barley). In the leaf  
 238 model only, we set the rate of P entry from the xylem ( $F(c, z, t)$ , Eqn. 10) proportional to the  
 239 experimental plant P uptake to simulate a representative plant P root uptake. We then combine the  
 240 models, i.e. let the rate of P entry from the xylem be estimated from the root model, and fit for the  
 241 remaining parameters ( $\gamma_c$  and  $\alpha_c$ ).

242 During the fitting process calibration step we minimise the sum of squares value between the  
 243 model's output for plant P uptake and leaf mass values against the experimental data values for  
 244 each the control and maximum applied P scenario (0 and 90/120 kg P ha<sup>-1</sup> respectively). With the  
 245 fitted parameters we then run the model for all applied P scenarios.

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246 The differences between modelling spring barley and winter barley are the time they are grown for  
247 (151 and 313 days, respectively), the initial P profile in the soil (20 mg P l<sup>-1</sup> decay profile and 16 mg P  
248 l<sup>-1</sup> constant profile, respectively) and leaf growth dependence (also depending upon air temperature  
249 for winter barley).

## 250 Results

251 We compare two sets of barley field experimental data against the coupled model, the leaf model  
252 (where plant P uptake is given by experimental data) and the root model. The aim is to address the  
253 differences between the models and how well they fit the experimental field data for barley.

254 First we compare the values for plant P uptake between the root and coupled model for spring  
255 barley at three different growth stages, GS31, GS45 and GS91 for seven applied P rates (0, 5, 10, 20,  
256 30, 60 and 90 kg P ha<sup>-1</sup>; Figure 2). The coupled model estimates higher plant P uptake compared to  
257 the root model, better fitting the experimental data; staying within one standard deviation except at  
258 high applied P rates (30, 60 and 90 kg P ha<sup>-1</sup> at GS31, 20, 60 and 90 kg P ha<sup>-1</sup> at GS45 and 30 and 60  
259 kg P ha<sup>-1</sup> at GS91). The feedback effect within the coupled model enables the root structure to  
260 become larger than in the root model and therefore the roots explore more of the soil and hence  
261 achieve an increased plant P uptake (Figure 8). The final model estimate (GS91) is more accurate  
262 than the earliest (GS31) due to not capturing the effects of possible lateral root proliferation due to  
263 higher applied P rates (Drew, 1975). Early differences are averaged out as the root system grows.

264 When considering plant P uptake in winter barley, the coupled model behaves similarly to the root  
265 model (Figure 3). At GS92, both models under-predict plant P uptake for the same reasons as stated  
266 in Heppell *et al.* (2015); the P profile is depleted which limits the amount of P available for uptake,  
267 and perhaps the total amount of P in the soil was different to that estimated by the one soil test for  
268 the whole site (Olsen P index 1). The effect of slow release P pools in the soil was not taken into  
269 consideration due to the fact experimental data for this phenomenon was not available.

270 By coupling the root model with the leaf model we are able to compare measured leaf mass values  
271 against the coupled and leaf model only for both spring barley (Figure 4) and winter barley (Figure 5)  
272 for different applied P rates. The coupled model accurately predicts leaf mass at GS91 for spring  
273 barley, however it estimates a more average value for earlier growth stages; not distinguishing any  
274 differences between applied P rates. The large errors bars in the experimental leaf mass data are  
275 possibly due to field variation, making it hard to distinguish any differences between applied P rates,  
276 especially at later growth stages (the experimental differences are not statically significant). In  
277 addition, the variation in experimental plant P uptake values for GS31 is less than for GS91 (18% to  
278 24%), implying little correlation between early and late plant P uptake (adjusted  $r^2=0.4$ ). For winter  
279 barley, the coupled model is able to match leaf mass at GS39, but vastly underestimates leaf mass at  
280 GS92 due to underestimating plant P uptake as mentioned above. The leaf model fits well across all  
281 scenarios for spring and winter barley as it takes the known plant P uptake from the experimental  
282 data as an input.

283 The leaf model component allows us to estimate P (Figure 6) and C mass (Figure 7) in the above  
284 ground tissue over the growing period of the crop. The estimated P mass is higher in the leaf model  
285 compared to the coupled model for both spring and winter barley. The estimated C mass is higher in  
286 the leaf model compared to the coupled model for winter barley, but the other way around for  
287 spring barley. In the winter barley case, the increased C and P masses in the leaf model are due to  
288 higher plant P uptake values (Figure 3 compared to Figure 2) resulting in a larger end leaf mass. For  
289 spring barley, C mass in the coupled model begins lower and ends higher compared to the leaf  
290 model because plant P uptake by the root system also begins lower and ends higher (P uptake  
291 remains constant in the leaf model). The sudden decrease in C and P mass, for winter barley, around  
292 the 250 day mark is due to the enforced halting of the root growth rate.

293 The root growth rate is affected by C mass (spring barley) and also temperature (winter barley);  
294 therefore different final root lengths can be observed between model simulations (Figure 8). The

leaf model created a longer root length compared to the coupled model in the winter barley scenario due to the early differences in C mass. For spring barley, the early C mass values for the coupled and leaf model were similar resulting in almost identical root growth rates and hence final root lengths. As C mass increases above a certain value any differences are masked when affecting the root growth rate. There was little difference in root length between the two different fertiliser applications (0 and 90/120 kg P ha<sup>-1</sup>), the largest being between the coupled model for winter barley GS92. Due to the small increase in plant P uptake between scenarios (0 and 120 kg P ha<sup>-1</sup>) there was little effect on increasing root length via the slow feedback loop created by the addition of the leaf model. Chemotropism effects from adding large amounts of P fertiliser could perhaps explain any differences between plant P uptake values at early growth stages. In the winter barley scenario, as root growth rate was dependent upon temperature, we see periods of no root growth matching periods of low temperature, as expected.

Heppell *et al.* (2015) considered the effects of discrete placing of fertiliser within the root zone against incorporating fertiliser throughout the soil for a range of cultivation options (mix 25, 20 and 10 cm, inverted plough, minimum tillage and no cultivation) for winter barley at GS92. We do the same in this paper for the new coupled model (Figure 9). We arrive at the same overall conclusion, placing fertiliser rather than incorporating achieves a higher plant P uptake estimate and under a wet climate (x5 flux of water at soil surface), such as in the UK, this difference decreases (9.9% to 0.3% and 9.8% to 4.5%) over no cultivation for a dry and wet climate respectively. Ploughing was also the best cultivation option moving top soil P to a lower depth, making it more accessible to a comparatively larger root system.

## Discussion

In order to obtain a more accurate representation of the growth of barley throughout a crop life cycle we have combined a below ground root-soil model with an above ground leaf model. By combining the two models we are able to let an above ground process (photosynthesis) affect a

below ground process (root growth) and vice versa. C is created via photosynthesis in the leaf model (dependent upon leaf mass and P) and stimulates root growth; increased root growth increases plant P uptake and hence leaf mass. This positive feedback effect could explain why crops with early plant P uptake levels grow more vigorously and can produce higher yields (Brenchley, 1929; Boatwright and Viets, 1966; Green *et al.*, 1973; Grant *et al.*, 2001). Due to possible unfavourable (e.g. dry) weather conditions, maximising early plant P uptake through greater root proliferation is also a good strategy to help ensure continuing capture of soil resources at later stages of growth.

From the modelling work conducted we can postulate that the whole crop model accurately estimates leaf mass at all growth stages given it has accurate estimates of plant P uptake (an average difference of 4.6% for the whole crop model for leaf mass, compared to 15.8% when using values one standard deviation away from the experimental data). Using the calibrated whole crop model we found the optimal fertiliser and cultivation scenario is to use a plough and place the P fertiliser. The largest increase in plant P uptake when placing fertiliser over incorporating fertiliser was 9.6% (plough, dry climate). The difference between incorporating and placing has been long studied and depends upon a range of criteria such as soil P concentration, soil temperature, crop species and price (Devine *et al.*, 1964; Mahler, 2001). Owusu-Gyimah *et al.* (2013) found that applying fertiliser at a depth of 10 cm and 20 cm away from the plant (placed P) gave the best outcome for maize growing under tropical conditions. By placing fertiliser instead of incorporating it throughout the soil the available P is being put where the root system is going to grow hoping to ensure early plant P uptake and a more successful crop. Hence Wager *et al.* (1986) found that P fertilizer application rates could be halved by placing fertiliser instead of incorporation because the applied P was more efficiently used. However, optimal fertiliser and cultivation methods depend on the initial soil P condition/distribution (Randall and Hoeft, 1988); this includes at the depth at which existing P is initially available within the soil (Heppell *et al.*, 2015).



344 For modelling across countries it will be important to measure soil available P levels consistently, by  
345 either using a common method or a set of common descriptors. Although, an international 'standard'  
346 soil extraction method is not necessarily needed; rather employing a basic soil property (e.g.  
347 sorption/buffer capacity) would be better to calibrate fertiliser recommendations. Modelling is the  
348 most appropriate way to overcome the problems of site specificity in soil P supply that confound  
349 current soil P test methods which do not apply to all soil types, i.e. across countries. Countries  
350 generally adopt a particular standard method for soil P tests; many different extractants are used.  
351 However, these do not necessarily give correlated results, for example across European laboratories  
352 (Neyroud and Lischer, 2002; Jordan-Meille *et al.*, 2012). It is possible that a more robust soil test will  
353 be developed in the future, that more accurately reflects immediate P availability to roots across  
354 different soil types. For example, using Diffusive Gradient in Thin films (DGT) based on soil P  
355 diffusion rates (Van Rotterdam *et al.*, 2009; Tandy *et al.*, 2011) or a method that mimics root P  
356 acquisition traits (De Luca *et al.*, 2015). The use of more mechanistic approaches to calculate soil  
357 available P levels via a more standardised test, or a combination of tests, enhances their applicability  
358 across a wider variety of soil types and may lead to more accurate assessment of fertiliser needs  
359 (Van Rotterdam *et al.*, 2014). Also, given that patterns of P concentration with depth in soil profiles  
360 vary between sites (Jobbágy and Jackson, 2001), it may also be important to assess surface  
361 stratification in no-tilled soils or in subsoils. Over-fertilising soils due to inaccurate estimation of  
362 requirement, or mis-interpretation of soil P supply through inappropriate tests leads not only to  
363 waste of finite reserves of phosphate-rock but also increased risk of P loss to water causing  
364 eutrophication (Hooda *et al.*, 2001). By using knowledge about the distribution of P within the soil  
365 and by modelling its implications, it should be possible to save on fertiliser costs by implementing  
366 better optimised treatments through targeting P use (Yang *et al.*, 2013; Withers *et al.*, 2014).  
367 Furthermore, since crop and fertiliser management have long-term effects on topsoil and subsoil P  
368 availability (Bolland and Baker, 1998), it will be important to validate the model over several years if  
369 it is to improve on current simpler approaches to decision making. Additional model features would

370 be needed, such as effects between cropping seasons, but would make for a more overall  
371 accomplished model. We note that the model would have to be calibrated separately for different  
372 crops.

373 Although there was little response to P application observed in the field trial in terms of plant P  
374 uptake at late growth stages (GS91 for spring barley and GS92 for winter barley), there was a  
375 response at early growth stages (GS31 for spring barley and GS39 for winter barley). This early  
376 response could imply that there were limiting environmental factors beyond nutritional inputs. Cold  
377 and dry conditions in spring are known to inhibit the transport of P from the soil to the root (Grant  
378 *et al.*, 2001). However, if the measured 'low' P soil was an underestimation for the total amount of  
379 available P in the soil then this could explain the lack of response at harvest observed in the field. In  
380 addition, field variation could in part explain the early response to applied P; however as the root  
381 system became larger during the latter growth stages any difference in plant P uptake and resulting  
382 yield was evened out. Due to the complex nature of cereal physiology (Sylvester-Bradley *et al.*, 2008),  
383 an early plant P uptake response does not necessarily indicate a higher final plant P uptake and yield;  
384 because the plant compensates by taking up more P later on as temperatures warm up. The slow  
385 feedback effect is a good explanation of the long term behaviour of the crop, and estimation of total  
386 plant P uptake.

387 Potentially, new ways to improve efficiency use of P can now be developed by combining recent  
388 advances in application technology, sensing technology, geo-spatial information and modelling so as  
389 to apply P where it is needed and importantly not apply it where it is not needed. Precision farming  
390 equipment is being widely adopted; now, its effective deployment depends on whether the vast  
391 amount of data available about a given plot of land can be interpreted to improve the precision and  
392 decrease the risks compared to current decision making (Sylvester-Bradley *et al.*, 1999). For example,  
393 soil nutrient maps, past yield maps, soil and canopy sensors and climate predictions may provide  
394 input data for integrated crop models to output quantitative predictions of fertiliser requirements so

395 that application as sowing can be adjusted in real time. However, the more immediate and  
396 preliminary prospect is of using simulation models to compare scenarios of possible treatments, to  
397 help guide future soil and fertiliser management strategies, and to accompany continuing field  
398 testing.

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#### 405 **References**

- 406 **Barber S (1984)** Soil nutrient bioavailability: a mechanistic approach. Wiley-Interscience.
- 407 **Bhadoria P, Kaselowsky J, Claassen N, Jungk A (1991)** Soil Phosphate Diffusion Coefficients: Their  
408 Dependence on Phosphorus Concentration and Buffer Power. *Soil Science of America Journal*  
409 **55**(1):56-60.
- 410 **Blackmore S (2014)** Address to the Oxford Farming Conference. 8 January 2014. See  
411 <http://www.ofc.org.uk/videos/2014/vision-farming-robots-2050> last accessed 07/09/2015.
- 412 **Boatwright G, Viets F (1966)** Phosphorus absorption during various growth stages of spring wheat  
413 and intermediate wheatgrass. *Agronomy Journal* **58**:185–188.
- 414 **Bolland M, Baker M (1998)** Phosphate applied to soil increases the effectiveness of subsequent  
415 applications of phosphate for growing wheat shoots. *Australian Journal of Experimental*  
416 *Agriculture* **38**(8):865-869.

417 **Brenchley W (1929)** The phosphate requirement of Barley at different periods of growth. *Annals of*  
418 *Botany* **43**:89–112.

419 **Brown L (2012)** Outgrowing the Earth: The Food Security Challenge in an Age of Falling Water Tables  
420 and Rising Temperatures. Taylor & Francis. ISBN 1-84407-185-5.

421 **Conway G, Barbier E (1990)** After the Green Revolution: Sustainable Agriculture for Development.  
422 Earthscan, London, UK. ISBN: 978-1-84971-930-8.

423 **Cordell D, Drangert J, White S (2009)** The story of phosphorus: Global food security and food for  
424 thought. *Global Environmental Change* **19**:292-305.

425 **Defra (2010)** Fertiliser Manual (RB209), 8<sup>th</sup> Edition. The Stationary Office, London.

426 **De Luca T, Glanville H, Harris M, Emmett B, Pingree M, De Sosa L, Morena C, Jones D (2015)** A novel  
427 rhizosphere trait-based approach to evaluating soil phosphorus availability across complex  
428 landscapes. *Soil Biology and Biochemistry* **88**:110-119.

429 **Déry P, Anderson B (2007)** Peak phosphorus. In: Energy Bulletin, 08/13/2007. Post Carbon Institute.  
430 Available: [energybulletin.net/node/33164](http://energybulletin.net/node/33164).

431 **Devine JJ, Gilkes R, Holmes MRJ (1964)** Field experiments on the phosphate requirements of Spring  
432 wheat and barley. *Experimental Husbandry* **11**:88-97.

433 **Drew MC (1975)** Comparison of the effects of a localised supply of phosphate, nitrate, ammonium  
434 and potassium on the growth of the seminal root system, and the shoot in Barley. *New*  
435 *Phytologist* **75**:479-490.

436 **Dunbabin V, Postma J, Schnepf A, Pagès L, Javaux M, Wu L, Leitner D, Chen Y, Rengel Z, Diggle A**  
437 **(2013)** Modelling root-soil interactions using three-dimensional models of root growth,  
438 architecture and function. *Plant and soil* **372**:93-124.

439 **Foyer C, Spencer C (1986)** The relationship between phosphate status and photosynthesis in leaves.  
 440 *Planta* **167**(3):369-375.

441 **George T, Brown L, Newton A, Hallett P, Sun B, Thomas W, White P (2011)** Impact of soil tillage on  
 442 the robustness of genetic component of variation in phosphorus (P) use efficiency in barley  
 443 (*Hordeum vulgare* L.) *Plant and Soil* **339**:113-123.

444 **Grant C, Flaten D, Tomasiewicz D, Sheppard S (2001)** The importance of early season phosphorus  
 445 nutrition. *Canadian Journal of Plant Science* **81**:211–224.

446 **Green D, Ferguson W, Warder F (1973)** Accumulation of toxic levels of phosphorus in the leaves of  
 447 phosphorus-deficient barley. *Canadian Journal of Plant Science* **53**:241–246.

448 **Greenwood D, Karpinets T, Stone D (2001)** Dynamic Model for the Effects of Soil P and Fertilizer P  
 449 on Crop Growth, P Uptake and Soil P in Arable Cropping: Model Description. *Annals of Botany*  
 450 **88**:279-291.

451 **Heppell J, Payvandi S, Zygalkis K, Smethurst J, Fliege J, Roose T (2014)** Validation of a spatial-  
 452 temporal soil water movement and plant water uptake model. *Geotechnique* **64**(7):526–539.

453 **Heppell J, Payvandi S, Talboys P, Zygalkis K, Fliege J, Langton D, Sylvester-Bradley R, Walker R,**  
 454 **Jones DL, Roose T (2015)** Modelling the optimal phosphate fertiliser and soil management  
 455 strategy for crops. *Plant and Soil special edition PSP5*. DOI: 10.1007/s11104-015-2543-0.

456 **Hooda P, Truesdale V, Edwards A, Withers P, Aitken M, Miller A, Rendell A (2001)** Manuring and  
 457 fertilization effects on phosphorus accumulation in soils and potential environmental implications.  
 458 *Advances in Environmental Research* **5**(1):13-21.

459 **Jeuffroy M, Vocanson A, Roger-Estrade J, Meynard J (2012)** The use of models at field and farm  
 460 levels for the ex ante assessment of new pea genotypes. *European Journal of Agronomy* **42**:68-78.

461 **Jobbágy E, Jackson R (2001)** The distribution of soil nutrients with depth: Global patterns and the  
462 imprints of plants. *Biogeochemistry* **53**(1):51-77.

463 Jones J, Hoogenboom G, Porter C, Boote K, Batchelor W, Hunt L, Wilkens P, Singh U, Gijsman A,  
464 Ritchie J (2003) The DSSAT cropping system model. *European Journal of Agronomy* **18**:235-265.

465 **Jordan-Meille L, Rubæk G, Ehlert P, Genot V, Hofman G, Goulding K, Recknagel J, Provolò G,**  
466 **Barracclough P (2012)** An overview of fertilizer – P recommendations in Europe: soil testing,  
467 calibration and fertilizer recommendations. *Soil Use and Management* **28**(4):419-435.

468 Keating B, Carberry P, Hammer G, Probert M, Robertson M, Holzworth D, Huth N, Hargreaves J,  
469 Meinke H, Hochman Z, McLean G, Verburg K, Snow V, Dimes J, Silburn M, Wang E, Brown S,  
470 Bristow K, Asseng S, Chapman S, McCown R, Freebairn D, Smith C (2003) An overview of APSIM,  
471 a model designed for farming systems simulation. *European Journal of Agronomy* **18**(3-4):267-  
472 288.

473 **Kuchenbuch RO, Buczko U (2011)** Re-visiting potassium- and phosphate-fertilizer responses in field  
474 experiments and soil-test interpretations by means of data mining. *Journal of Plant Nutrition and*  
475 *Soil Science* **174**:171-185.

476 **Lohry R (1998)** Surface Banding superior To Broadcasting on Reduced-Till. *Fluid Journal* **22**:14-18.

477 **Lynch J (2007)** Roots of the second green revolution. *Australian Journal of Botany* **55**:493-512.

478 **Mahler R (2001)** Fertilizer Placement. CIS. Soil Scientist, Department of plant, Soil, and  
479 Entomological Sciences, University of Idaho.

480 **Neyroud J, Lischer P (2002)** Do different methods used to estimate soil phosphorus availability  
481 across Europe give comparable results? *Journal of Plant Nutrition Soil Science* **166**(4):422-431.

482 **Nye P, Tinker P (1977)** Solute movement in the soil-root system. Blackwell Science Publishers.

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483 **Owusu-Gyimah V, Nyatuame M, Ampaw F, Ampadu P (2013)** Effect of depth and Placement  
 484 Distance of Fertilizer NPK 15-15-15 on the Performance and Yield of Maize Plant. *International*  
 485 *Journal of Agronomy and Plant Production*. **4**(12):3197-3204.

486 **Randall G, Hoefl R (1988)** Placement Methods for Improved Efficiency of P and K Fertilizers: A  
 487 Review. *Journal of Production Agriculture* **1**(1):70-79.

488 **Roose T, Fowler A, Darrah P (2001)** A mathematical model of plant nutrient uptake. *Mathematical*  
 489 *Biology* **42**(4):347–360.

490 **Roose T, Flower A (2004a)** A model for water uptake by plant roots. *Journal of Theoretical Biology*  
 491 **288**:155-171.

492 **Roose T, Fowler A (2004b)** A mathematical model for water and nutrient uptake by plant root 567  
 493 systems. *Journal of Theoretical Biology* **288**:173-184.

494 **Selmants P, Hart S (2010)** Phosphorus and soil development: Does the Walker and Syers model  
 495 apply to semiarid ecosystems? *Ecology* **91**(2):474-484.

496 **Sylvester-Bradley R (1991)** Modelling and mechanisms for the development of agriculture. Aspects  
 497 of Applied Biology 26, The Art and Craft of Modelling in Applied Biology, 55-67.

498 **Sylvester-Bradley R, Lord El, Sparkes D, Scott RK, Wiltshire JJ, Orson JO (1999)** An analysis of  
 499 precision farming in Northern Europe. *Soil Use and Management* **15**:1-9.

500 **Sylvester-Bradley R, Berry P, Blake J, Kindred D, Spink J, Bingham I, McVittie J, Foulkes J (2008)** The  
 501 wheat growth guide, Spring 2008, 2<sup>nd</sup> edition. HGCA, London 30pp  
 502 <http://www.hgca.com/media/185687/g39-the-wheat-growth-guide.pdf>, last accessed  
 503 12/09/2014.

504 **Tandy S, Mundus S, Yngvesson J, de Bang T, Lombi E, Schjoerring J, Husted S (2011)** The use of DGT  
505 for prediction of plant available copper, zinc and phosphorus in agricultural soils. *Plant and Soil*  
506 **346**(1-2):167-180.

507 **Thornley JH (1995)** Shoot: root allocation with respect to C, N and P: an investigation and  
508 comparison of resistance and teleonomic models. *Annals of Botany* **75**(4):391-405.

509 **Vaccari D (2009)** Phosphorus: a looming crisis. *Scientific American* **300**:54-49.

510 **Valkama E, Uusitalo R, Turtola E (2011)** Yield response models to phosphorus application: a  
511 research synthesis of Finnish field trials to optimize fertilizer P use of cereals. *Nutrient Cycling in*  
512 *Agroecosystems* **91**:1–15.

513 **Van Genuchten M (1980)** A closed-form equation for predicting the hydraulic conductivity of  
514 unsaturated soil. *Soil Science Society of America Journal*, **44**(5):892-898.

515 **Van Rotterdam A, Temminghoff E, Schenkeveld W, Hiemstra T, Riemsdijk W (2009)** Phosphorus  
516 removal from soil using Fe oxide-impregnated paper: Processes and applications. *Geoderma*  
517 **151**(3-4):282-289.

518 **Van Rotterdam A, Bussink D, Reijneveld J (2014)** Improved Phosphorus Fertilisation Based on Better  
519 Prediction of Availability in Soil. International Fertiliser Society, Proceeding 755, ISBN 978-0-  
520 85310-392-9.

521 **Vance C, Uhde-Stone C, Allan D (2003)** Phosphorus acquisition and use: critical adaptations by  
522 plants for securing a non-renewable resource. *New Phytologist* **157**:423-447.

523 **Wager BI, Stewart JWB, Henry JL (1986)** Comparison of single large broadcast and small annual  
524 seed-placed phosphorus treatments on yield and phosphorus and zinc contents of wheat on  
525 Chernozemic soils. *Canadian Journal of Soil Science* **66**:237–248.



526 **Wissuwa M, Gamat G, Ismail AM (2005)** Is root growth under phosphorus deficiency affected by  
527 source or sink limitations? *Journal of experimental botany* **56**(417):1943-1950.

528 **Withers PJA, Sylvester-Bradley R, Jones DL, Healey JR, Talboys PJ (2014)** Feed the crop not the soil:  
529 rethinking phosphorus management in the food chain. *Environmental Science and Technology*  
530 **48**:6523-6530.

531 **Withers PJA, van Dijk KC, Neset T-SS, Nesme T, Oenema O, Rubæk GH, Schoumans OF, Smit B,**  
532 **Pellerin S (2015)** Stewardship to tackle global phosphorus inefficiency: The case of Europe.  
533 *AMBIO* 44 (Suppl.):S193-S206.

534 **Yang X, Post W, Thornton P, Jain A (2013)** The distribution of soil phosphorus for global  
535 biogeochemical modeling. *Biogeosciences Discussions* **9**(11):16347-16380.

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567 and either no fertiliser, 90 kg P ha<sup>-1</sup> incorporated or 90 kg P ha<sup>-1</sup> placed) for a normal climate and a  
568 wetter climate [\(x5 flux of water at soil surface\)](#).

569 **List of Tables**

570 Table 1: A list of the parameters used for the 3 models: leaf, root and coupled.

| Parameter   | Definition                              | Value  | Units  |
|---|---|--|--|
| <b>Leaf Model (Values from Thornley, J. H., 1995)</b> |   |  |  |
| $k_G$   | Leaf growth rate constant               | 1000   | $\left(\frac{\text{kg C}}{\text{kg Leaf}} \frac{\text{kg P}}{\text{kg Leaf}} \text{day}\right)^{-1}$ |
| $k_{litt}$  | Leaf litter rate constant               | 0.05   | $\text{day}^{-1}$  |
| $k_{mlitt}$   | Leaf litter Michealis-Menten constant   | 0.5  | kg Leaf  |
| $k_C$   | Photosynthesis constant                 | 0.1  | $\frac{\text{kg C}}{\text{kg Leaf}} \text{day}^{-1}$   |
| $k_M$   | Leaf self-shading constant              | 1  | kg Leaf  |
| $J_C$   | Carbon product inhibition constant      | 0.1  | $\frac{\text{kg C}}{\text{kg Leaf}}$   |
| $f_c$   | Fraction of C used for leaf growth      | (fitted)   | $\frac{\text{kg C}}{\text{kg Leaf}}$   |
| $f_p$   | Fraction of P used for leaf growth      | (fitted)   | $\frac{\text{kg P}}{\text{kg Leaf}}$   |
| $k_1$   | P used for photosynthesis               | (fitted)   | $\frac{\text{kg Leaf}}{\text{kg P}}$   |
| $k_p$   | P:C ratio for photosynthesis production | 0.005-0.05   | $\frac{\text{kg P}}{\text{kg C}}$  |
| $\beta_p$   | Rate of P output to phloem              | 0  | $\frac{\text{kg Leaf}}{\text{day}}$  |
| $F$   | Rate of P entry from xylem              | Taken from barley experimental data or root model output | $\frac{\text{kgP}}{\text{day}}$  |
| $A_T$   | Air temperature                         | Taken from Local   | $^{\circ}\text{C}$   |

|  |   |                              |  |
|--|---|------------------------------|--|
|  |   | Met office MIDAS<br>stations |  |
| $\beta_c$  | Rate of C output to phloem                              | (fitted)                     | $\frac{\text{kgL}}{\text{day}}$            |
| $s_1$  | Air temperature slope constant                          | (fitted)                     | -  |
| $s_2$  | Air temperature transition constant                     | (fitted)                     | $^{\circ}\text{C}$                         |
| <b>Root-Soil Model (values from Heppell <i>et al.</i>, 2015)</b> |   |                              |  |
| $D_0$  | Water diffusivity                                       | $10^3$                       | $\text{cm}^2 \text{day}^{-1}$              |
| $K_s$  | Water hydraulic conductivity                            | 5                            | $\text{cm}^2 \text{day}^{-1}$              |
| $D_f$  | P diffusivity in free water                             | $10^{-5}$                    | $\text{cm}^2 \text{day}^{-1}$              |
| $d$  | Impedance factor  | 2                            | -  |
| $a$  | Main order root radius                                  | 0.085                        | cm   |
| $a_1$  | first order root radius                                 | 0.060                        | cm   |
| $k_r$  | Root radial water conductivity                          | $7.85 \cdot 10^{-6}$         | $\text{m}^2 \text{s}^{-1} \text{MPa}^{-1}$ |
| $k_z$  | Root axial hydraulic conductivity                       | $1.198 \cdot 10^{-2}$        | $\text{m}^4 \text{Pa}^{-1} \text{s}^{-1}$  |
| $\psi_1$   | Density of first order roots                            | 2.33                         | $\text{cm}^{-1}$                           |
| $p_r$  | Root internal xylem pressure                            | 1                            | Pa   |
| $p_c$  | Characteristic suction pressure                         | 0.0232                       | MPa  |
| $L_0$  | Max length of main order root                           | 150                          | cm   |
| $L_1$  | Max length of first order root                          | 7.9                          | cm   |
| $L$  | Root tip position                                       | $0-L_0$                      | cm   |
| $b$  | Buffer power  | 23.28                        | -  |
| $\theta$   | Angle between the main root and<br>first order branches | 60                           | degrees                                    |
| $\phi$   | Porosity of soil  | 0.3                          | -  |
| $p_r^0$  | Initial root internal xylem pressure                    | 1                            | Pa   |

|                      |   |                      |   |
|----------------------|---|----------------------|---|
| $\lambda_1$          | Root internal xylem pressure<br>parameter                 | $2.7 \cdot 10^{-3}$  | Pa/ degC  |
| $\lambda_2$          | Root internal xylem pressure<br>parameter                 | $8.46 \cdot 10^{-4}$ | Pa/% humidity   |
| $\lambda_3$          | Root internal xylem pressure<br>parameter                 | $7.9 \cdot 10^{-2}$  | Pa  |
| $\delta$             | Flux of water parameter                                   | $2.69 \cdot 10^{-2}$ | -   |
| $\alpha$             | Flux of water parameter                                   | $1.2 \cdot 10^{-6}$  | $\text{m s}^{-1}$ of water                              |
| $\beta$              | Flux of water parameter                                   | $2.22 \cdot 10^{-6}$ | $\text{m s}^{-1}$ of water/degC                         |
| $\gamma$             | Flux of water parameter                                   | $5.35 \cdot 10^{-4}$ | $\text{m s}^{-1}$ of water/ $\text{m s}^{-1}$ of<br>air |
| $E$                  | Flux of water parameter                                   | $5 \cdot 10^{-4}$    | $\text{m s}^{-1}$ of water                              |
| $l_w$                | Bottom of the soil  | 200                  | cm  |
| <b>Coupled Model</b> |   |                      |   |
| $\gamma_c$           | Root carbon growth parameter                              | (fitted)             | -   |
| $\alpha_c$           | Strength of carbon effect on root<br>growth               | (fitted)             | -   |
| $A$                  | Strength of temperature<br>dependence on root growth rate | 0.0780               | -   |

571

572 **Table 2: The fitted parameter set for the leaf and coupled models, for spring barley and winter**  
573 **barley.**

| Parameter  |           | Value for Spring barley | Value for winter barley |
|------------|-----------|-------------------------|-------------------------|
| Leaf Model | $\beta_c$ | 0.0001                  | 0.0001                  |

|               |            |                      |                      |
|---------------|------------|----------------------|----------------------|
|               | $k_1$      | 100                  | 859                  |
|               | $f_c$      | 0.5                  | 0.5                  |
|               | $f_p$      | $7 \cdot 10^{-4}$    | $1.6 \cdot 10^{-3}$  |
|               | $s_1$      | n/a                  | 20.78                |
|               | $s_2$      | n/a                  | -1.446               |
| Coupled model | $\gamma_c$ | $1.30 \cdot 10^{-5}$ | $1.31 \cdot 10^{-4}$ |
|               | $\alpha_c$ | 1                    | 1.982                |